Examples of the Use of Linear Programming for Problem-solving in Healthcare Management

Exemplos de Uso de Programação Linear para Resolução de Problemas na Área de Gestão em Saúde
Ejemplos del Uso de Programación Lineal para la Resolución de Problemas en la Gestión Sanitaria

RESUMO

Objetivo: Demonstrar como a Pesquisa Operacional, e em particular a Programação Linear, oferece um conjunto robusto de ferramentas quantitativas para a otimização de processos em gestão na área da saúde. **Método:** Através de três exemplos práticos e realistas — a alocação de enfermeiros em turnos hospitalares, a otimização do acesso geográfico a serviços especializados e aumento da sensibilidade diagnóstica de serviços laboratoriais, — exploramos a formulação de modelos, a interpretação de resultados e as limitações inerentes à técnica. **Resultados:** O trabalho conclui que, embora a PL seja uma ferramenta poderosa, seu valor é maximizado quando combinada com o conhecimento técnico dos gestores e uma compreensão clara de suas limitações, ressaltando a necessidade de capacitação contínua para a aplicação eficaz de modelos de otimização.
DESCRITORES: Pesquisa Operacional, Programação Linear, Alocação de Recursos, Administração Hospitalar.

ABSTRACT

Objective: To demonstrate how Operations Research , and specifically Linear Programming, provides a robust set of quantitative tools for process optimization in healthcare management. **Method:** Through three realistic practical examples—nursing shift scheduling, the optimization of geographic access to specialized services, and the enhancement of diagnostic sensitivity in laboratory services—we explore model formulation, results interpretation, and the inherent limitations of the technique. **Results:** The study concludes that while LP is a powerful tool, its value is maximized when combined with the technical expertise of managers and a clear understanding of its limitations, highlighting the necessity of continuous training for the effective application of optimization models.
DESCRIPTORS: Operations Research, Linear Programming, Resource Allocation, Hospital Administration**.**

RESUMEN

Objetivo: Demostrar cómo la Investigación Operativa, y en particular la Programación Lineal, ofrece un conjunto robusto de herramientas cuantitativas para la optimización de procesos en la gestión sanitaria. **Método:** A través de tres ejemplos prácticos y realistas —la asignación de enfermeros en turnos hospitalarios, la optimización del acceso geográfico a servicios especializados y el aumento de la sensibilidad diagnóstica en servicios de laboratorio— exploramos la formulación de modelos, la interpretación de resultados y las limitaciones inherentes a la técnica. **Resultados:** El trabajo concluye que, si bien la PL es una herramienta poderosa, su valor se maximiza cuando se combina con el conocimiento técnico de los gestores y una comprensión clara de sus limitaciones, resaltando la necesidad de una capacitación continua para la aplicación eficaz de los modelos de optimización.
DESCRIPTORES: Investigación Operativa, Programación Lineal, Asignación de Recursos, Administración Hospitalaria.

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INTRODUCTION

The management of healthcare systems, whether public or private, operates in a highly complex environment characterized by finite resources, multiple objectives, and a constant need to improve the quality and efficiency of care. In this scenario, decision-making based purely on intuition or traditional trial-and-error methods becomes insufficient [1]. Operations Research (OR) emerges as an applied science dedicated to providing an analytical basis for more effective decisions, utilizing an arsenal of mathematical, statistical, and computational methods [2].

Among OR tools, Linear Programming (LP) stands out for its versatility and power in solving optimization problems. LP has been successfully applied in various industries to minimize costs and maximize productivity, and its potential in healthcare is equally vast [3]. Studies demonstrate its utility in a variety of contexts, ranging

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from diet planning and the optimization of radiation therapy treatments to large-scale resource allocation, such as surgery scheduling, hospital bed management, and the optimization of work schedules for healthcare professionals ( ) [(4,5)] . In Brazil, research has already explored the use of LP to optimize geographic access to healthcare networks within the Unified Health System (SUS) and to evaluate the performance of primary care services [(6,7)] .

However, the mere presentation of theoretical models is insufficient. The successful application of LP requires a deep understanding not only of how to formulate a problem but also of how to interpret its results, recognize its limitations, and navigate the practical challenges of its implementation. This article aims to fill this gap by offering an in-depth and critical analysis of the application of Linear Programming in healthcare. We will use practical and more realistic examples to illustrate the entire process: from modeling to the discussion of sensitivity analysis and the need for extensions such as Integer Programming. The ultimate goal is to empower health managers and analysts to view LP not as a mathematical "black box," but as a strategic and transparent tool for improving decision-making.

METHOD

This is an applied and quantitative study, characterized by mathematical modeling and scenario simulation. The research did not involve the collection of primary data from human subjects, relying instead on technical parameters extracted from specialized literature and health regulatory standards to construct realistic models.

The study was conducted between February 2025 and December 2025. The simulated scenarios were constructed based on the context of the Unified Health System (SUS), considering specific factors such as nursing labor legislation and the structure of the SUS healthcare networks.

The "sample" of this study consists of three fundamental problem types in health management, selected for their representativeness and complexity:

1. **Staffing:** Allocation of nursing shifts.
2. **Network management:** Optimization of geographic and budgetary access to physical therapy services.
3. **Diagnostic efficiency:** Maximizing laboratory sensitivity and cost-effectiveness. The inclusion criteria for selecting these problems were: (a) the linear nature of the constraints and objectives; (b) direct relevance to managerial decision-making; and (c) availability of technical parameters for modeling

Linear Programming is a mathematical technique for optimizing an outcome (e.g., minimizing a cost or maximizing service) given certain constraints (such as budget, time, or capacity). Every LP problem consists of three essential elements:

1. Decision Variables: These represent the quantities to be determined. They are the "levers" that the manager can adjust. For example, the number of nurses to be allocated to each shift.

2. Objective Function: This is the mathematical expression, in terms of the decision variables, that one wishes to optimize (maximize or minimize). It must be a linear function.

3. Constraints: These are linear equations or inequalities that limit the values the decision variables can take, representing real-world constraints (staff availability, budget, minimum demand, etc.).

Once a problem is formulated, the Simplex Method, developed by George Dantzig in 1947, is the algorithm most commonly used to solve it [(8)] . Although the mathematical details are complex, the intuition behind the method is geometric and elegant. The set of all solutions that satisfy the constraints is called the "feasible region." In a problem with two variables, this region is a polygon in the plane. The Simplex Method demonstrates that the optimal solution (if it exists) will always lie at one of the vertices (or "corners") of this polygon. The algorithm works intelligently, starting from an initial vertex and moving to adjacent vertices that improve the value of the objective function, until it is no longer possible to find a better vertex. Modern software, such as Excel's Solver or Python libraries (Scipy, PuLP), automates this process, enabling the solution of problems with thousands of variables and constraints.

To illustrate the application, we present two examples adapted from common challenges in healthcare management.

Example 1: Nurse Scheduling (Integer Programming)

Problem: A hospital needs to create the daily work schedule for a unit, minimizing personnel costs and ensuring minimum nursing coverage across different shifts. Labor laws require that each nurse work a continuous 8-hour shift. Based on historical data, the demand for nurses varies throughout the day, as shown in Table 1.

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Table 1: Minimum staffing requirements at specific times

Period	Time	Minimum Demand
1	6:00 a.m. to 10:00 a.m.	14
2	10:00 AM to 2:00 PM	8
3	2:00 PM to 6:00 PM	12
4	6:00 PM to 10:00 PM	6
5	10:00 PM to 6:00 AM	4

Source: Authors.

Decision Variables: Since the number of nurses must be an integer, we use Integer Programming, a variant of LP. The variables are:

•x_1: Number of nurses starting at 06:00 (working from 06:00 to 14:00)

•x_2: Number of nurses starting at 10:00 (working from 10:00 to 18:00)

•x_3: Number of nurses starting at 2:00 PM (working from 2:00 PM to 10:00 PM)

•x_4: Number of nurses starting at 6:00 PM (working from 6:00 PM to 2:00 AM the following day)

•x_5: Number of nurses starting at 10:00 PM (working from 10:00 PM to 6:00 AM the following day)

•Objective Function (Minimize the total number of nurses hired):

Minimize $Z = x_1 + x_2 + x_3 + x_4 + x_5$

•Constraints (Coverage in each period must meet demand):

•Period 1 (6:00 AM–10:00 AM): $x_1 \geq 14$ (Only the shift starting at 6:00 AM covers this period)

•Period 2 (10:00 AM–2:00 PM): $x_1 + x_2 \geq 8$ (Covered by those who started at 6:00 AM and 10:00 AM)

•Period 3 (2:00–6:00 PM): $x_2 + x_3 \geq 12$

•Period 4 (6:00 PM–10:00 PM): $x_3 + x_4 \geq 6$

•Period 5 (10 p.m.–6 a.m.): $x_4 + x_5 \geq 4$

•Non-negativity and Integers: x_i ≥ 0 and x_i must be an integer for all i.

Example 2: Optimizing Access to Physical Therapy Centers

Problem: A regional health department wishes to allocate its quota of 1,000 monthly physical therapy sessions between two new centers (Center A and Center B) to maximize the total number of patients served. Center A, the larger of the two, has a cost of R$ 50 per session and a maximum capacity of 700 sessions/month. Center B, the smaller of the two and located in a lower-cost area, has a cost of R$ 40 per session and a capacity of 500 sessions/month. The total budget available for the sessions is R$ 45,000.

•Decision Variables:

•x_A: Number of sessions allocated to Center A

•x_B: Number of sessions allocated to Center B

•Objective Function (Maximize the number of sessions):

Maximize Z = x_A + x_B

•Constraints:

•Budget: 50x_A + 40x_B ≤ 45,000

•Capacity of Center A: x_A ≤ 700

•Center B Capacity: x_B ≤ 500

•Total Quota: x_A + x_B ≤ 1000 (This constraint may be redundant, but it is good to state it explicitly)

•Non-negativity: x_A ≥ 0, x_B ≥ 0

Example 3: Maximizing the laboratory routine.

A certain laboratory performs two types of tests for the same purpose (Tests A and B), both of which complement each other and need to be repeated more than once for the best diagnostic predictability. Test A has a diagnostic sensitivity of 60% and Test B of 70%; the cost of A is R$ 5.00 and of B R$ 10.00.

Assuming that the laboratory needs to issue reports with the highest diagnostic accuracy (repeating the tests) within a maximum of 120 minutes at the best price, our objective is to minimize the cost of the tests and optimize the response in terms of report sensitivity and in the shortest possible time. Quality control requires that diagnostic sensitivity be at least 60%, and the turnaround time be 20 and 40 minutes,

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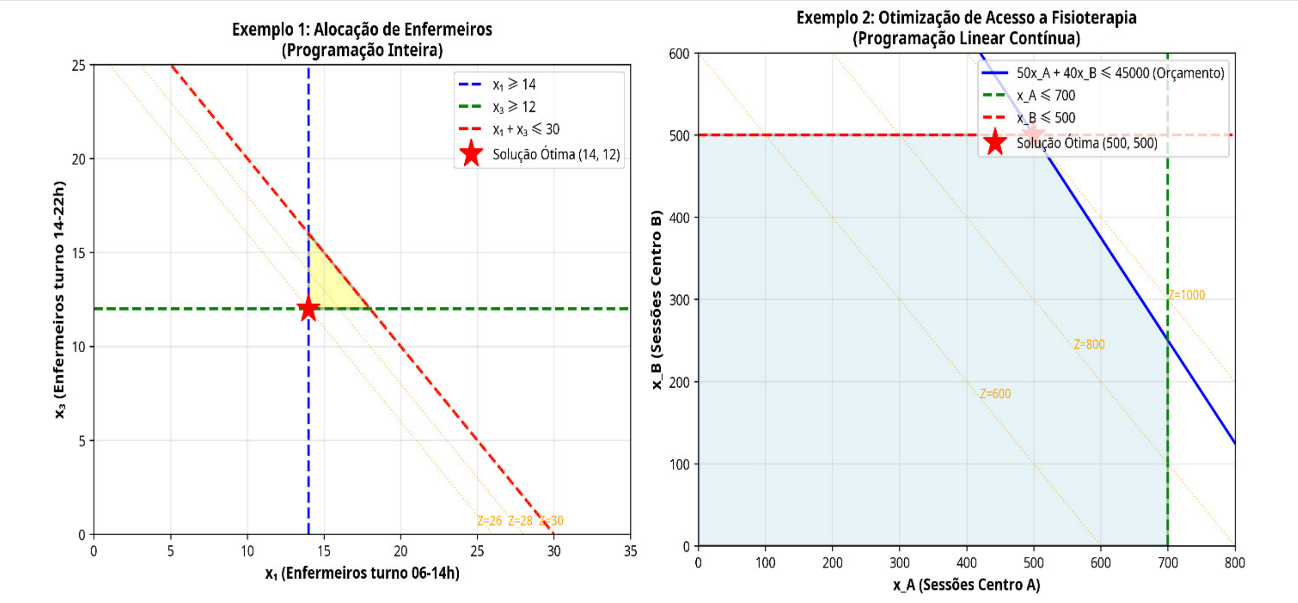
respectively.

RESULTS

The figure below presents the geometric representation of both examples, showing the feasible region (shaded area) and the optimal point (red star). In Example 1, the feasible region is defined by the minimum demand constraints, and the optimal solution lies at the vertex that minimizes the total number of nurses. In Example 2, the feasible region is a more complex polygon, bounded by the budget, center capacities, and total quota, with the optimal solution maximizing the number of sessions (Figure 1).

Figure 1: Graphical representations of the solutions to the problems described in Examples 1 and 2, using linear programming.



Source: Authors.

Solution to Example 1 (Nurse Allocation)

Solving the Integer Programming problem, the optimal solution is:

- x_1 = 14 (14 nurses starting at 6:00 AM)
- x_2 = 0 (No nurses starting at 10:00)
- x_3 = 12 (12 nurses starting at 2:00 PM)
- x_4 = 0 (No nurses starting at 6:00 PM)
- x_5 = 4 (4 nurses starting at 10:00 PM)

The minimum cost is 30 nurses in total (Z = 14 + 0 + 12 + 0 + 4).

Solution to Example 2 (Access to Physical Therapy)

This is a classic LP problem. The optimal solution is:

- x_A = 500 (500 sessions at Center A)
- x_B = 500 (500 sessions at Center B)

The maximum number of sessions is 1,000 (Z = 500 + 500). The total cost would be 50*500 + 40*500 = 25,000 + 20,000 = R$ 45,000, using the entire budget. Center A's capacity is not fully utilized, and the total quota is reached.

Solution to Example 3:

The equations relating to the constraints in Example 1 are: Execution time: x 1 + x 2 ≤ 120 Exam A: x 1 ≥ 20 Exam B: x 2 ≥ 40 Thus, the mathematical model is The optimal solution to the problem lies at the intersection of the lines ax 1=20 and bx 2=40. The optimal point of the problem is (x 1,x 2) = (20, 40), that is, four repetitions of exam A and four of exam B.

Using this methodology, we can verify the optimal hypothesis regarding the turnaround time for the reports and the minimum cost. A minimum cost of R$60.00 can be achieved if four repetitions of both exams are performed simultaneously.

DISCUSSION

In the example of the nurses, the solution of not hiring anyone for the 10 a.m. and 6 p.m. shifts may create bottlenecks or overload in adjacent shifts, even if the minimum demand is met. The model does not account for staff fatigue or flexibility. Sensitivity Analysis is a crucial step here. It answers questions such as: "To what extent can the cost of a shift increase before the optimal solution changes?"

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or "What is the impact on total cost if demand in period 3 increases by one unit?" This analysis provides the manager with an understanding of the solution's robustness and the system's most critical variables.

In the physical therapy example, the solution allocates as many sessions as possible (500) to the cheapest center (B) and the remainder to Center A until the budget is exhausted. What would happen if the budget were R$ 48,000? The solution would change to x_A = 700 and x_B = 300, showing that Center A's capacity becomes the active constraint. Sensitivity analysis allows the manager to understand the value of each resource. For example, every additional real in the budget (up to a certain point) can generate more sessions, and the analysis quantifies exactly that gain.

CONCLUSION

Linear Programming is a proven tool for addressing complex resource allocation challenges in the healthcare sector. As demonstrated, its application goes far beyond simple theoretical problems, informing strategic decisions regarding staffing schedules, service allocation, and budget utilization. However, this study sought to emphasize that the power of LP does not lie in the blind application of its algorithms, but in critical and informed use.

True value emerges when managers understand the model formulation, question its assumptions, perform sensitivity analyses, and are aware of its limitations. The decision to use Integer Programming, for example, reflects this deeper understanding. Successful implementation is not only a technical challenge but also a managerial one, requiring quality data, appropriate tools, and a culture that values evidence-based decision-making.

Decision-making in the healthcare sector opens the door to the application of optimization techniques in resource allocation problems, presenting itself as a complementary tool to economic evaluation models; statistics thus becomes an extremely necessary tool for use in healthcare.

Therefore, more than just a mathematical technique, Linear Programming should be viewed as a management philosophy: a structured approach to thinking about complex problems, identifying the key levers of decision-making, and finding, in a transparent and defensible manner, the best path forward. Investing in training healthcare professionals to use these tools is not a luxury, but a necessity for the sustainability and efficiency of future healthcare systems.

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